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ABSTRACT

This paper presents the background of the Project for the Mathematical Development of Children (PMDC), a list of nine objectives which direct PMDC's long-range activities, and a discussion of the rationale behind PMDC's approach. PMDC's activities are described in terms of the interviewing procedures used with children, teaching experiments and observational studies, methods of assessing children's knowledge, and dissemination of project information. (DT)

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NEW FRONTIERS IN MATHEMATICS EDUCATION:

PROJECT FOR THE MATHEMATICAL DEVELOPMENT OF CHILDREN

A presentation at the AAAS Annual Boston Meeting
February 23, 1976

by Eugene D. Nichols, Director - PMDC

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
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I. HISTORY

The decade of the 70's began with a reexamination of and debates about the virtues and shortcomings of the mathematics curricula produced during the 50's and 60's, called "modern math." During Fall 1972, the Mathematics Education faculty at the Florida State University began discussions concerning the future directions mathematics education might take. A conference of selected knowledgeable individuals who would reexamine the past and suggest an appropriate course for the future seemed to be in order.

The Florida State University provided financial support for a two-day conference which was held in Tallahassee on September 10-11, 1973. The purpose of the conference was to explore the major issues related to the teaching and learning of mathematics in the elementary grades. To accomplish this, sixteen individuals were invited to take part in the conference: eight individuals prepared a paper in advance and each of the remaining individuals prepared a reaction to one of the eight papers.

The authors of the papers, institutions, the titles of their papers, and the reactors to the papers were:

Stephen Willoughby, New York University, *"Lessons Learned and Pitfalls to be Avoided in the Continuing Curricular Reform Efforts in Mathematics Education."* Reactor: Edward Begle Stanford University.

Jack Downes, Emory University (presently at Georgia State University), *"What's the Go of That? What's the Particular Go of That?"* Reactor: Robert Kalin, Florida State University.

Robert Wirtz, Carmel, California, *"Toward an Unfinished Curriculum."* Reactor: William McKillip, University of Georgia.

Max Bell, University of Chicago, *"Some Needed Directions and Empahses in the Mathematical Education of Everyman in the*

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Pre-High School Years." Reactor: John LeBlanc, Indiana University.

Walter Dick, Florida State University, *"Role of Educational Technology in Mathematics Curriculum."* Reactor: Tom Denmark, Florida State University.

James Wilson, University of Georgia, *"The Evaluation Component of a Mathematics Curriculum Development."* Reactor: Gerald Rising, University of Buffalo.

Stanley Erlwanger, University of Illinois, *"The Observation-Interview Method and Some Case Studies."* Reactor: Leslie Steffe, University of Georgia.

Charles Smock, University of Georgia, *"Discovering Psychological Principles for Mathematics Instruction."* Reactor: Merlyn Behr, Northern Illinois University.

The publication, *Proceedings of the Conference on the Future of Mathematical Education* is available for $3.50 from: PMDC, Florida State University, Johnston Building, Tallahassee, FL 32306. Please make the check payable to Florida State University.

Even after the many discussions and deliberations which followed the conference, there was no obvious message that would suggest the appropriate emphases in the future mathematics curriculum. It seemed clear that the present knowledge about how children learn mathematics was not sufficient to serve as a guide for constructing a pedagogically sound mathematics curriculum. That is, a curriculum in which more students will succeed in learning the skills and concepts usually taught in the early grades.

During Fall 1973, a proposal was submitted to the National Science Foundation requesting financial support for a project in which principal investigators located at Florida State University, University of Georgia, University of Texas, and University of Ohio would work with first and second grade children. A grant for the initial phase to cover the period June 1974 - August 1975 was given to begin the Project for the Mathematical Development of Children (PMDC) with Dr. Eugene D. Nichols of Florida State University as Director. The second phase of the Project is now under way at only two sites, Florida State University and University of Georgia.

II. MAJOR CONCERNS

In the initial proposal PMDC was envisioned as directing its long-range activities toward the following nine objectives:

1. To develop interview techniques with individual children, which will result in insights into children's modes and patterns of thinking.
2. To work with teachers to develop techniques for evaluating individual children's understandings in mathematics, mainly through observations and interviews.
3. To develop and test techniques for reliably assessing the understandings and skills children have when entering the first and second grades.
4. To study the feasibility of teaching children some selected concepts and skills which are not ordinarily taught in standard curricula at this age level.
5. To study the feasibility of teaching the usual first and second grade level concepts and skills, but employing different approaches with the aim of achieving greater success.
6. To develop modules, including various aids, in pursuit of the above five objectives.
7. To identify those practices which teachers succeed with; to capture, refine and extend these practices to other teachers.
8. To explore ways in which teachers can successfully learn to incorporate into their daily practice the materials produced and findings obtained in meeting objectives one through seven.
9. To develop techniques and procedures for evaluating all activities pursued in meeting objectives one through eight.

Activities during the two years have been focused primarily on objectives 1, 3, 5, and 6.

III. RATIONALE

To propose that a child's thinking process serve as a basis for curriculum development is to presuppose that curriculum makers agree as to what this process is. This is not the case. Even if it were, it is further assumed that there is agreement on what implications this thinking process would have for the curriculum.

In the actual world of today's elementary school classroom, with not much hope for drastic changes for the better in the foreseeable future, it appears that to build a realistic, yet sound basis for the mathematics curriculum, the mathematical thoughts of children in their normal school habitat need to be studied intensively. Given an opportunity to think freely, children clearly display certain patterns of thought as they deal with the usual mathematical situations encountered daily in their classroom. A videotape record of the child's expressed thoughts, uninfluenced by any teaching on the part of the interviewer, provides a rich source for the formulation of conjectures as to what these thoughts are, what mental structures the child has developed, and how the child uses these structures when dealing with the ordinary concepts of arithmetic. An analysis of this videotape record further prompts some conjectures about possible sources of what the adult views as "misconceptions," about how the school environment (the teacher and/or the materials) influences the child's thinking, and about whether the teacher and/or the materials "fights" the child's natural thought processes.

In response to the above rationale, the activities in PMDC focus on understanding the child's thinking within the context of the mathematical environment found in the schools.

IV. PMDC's ACTIVITIES

A. Interviews

First and second grade children are interviewed by principal investigators; the interviews are videotaped. The videotapes are then studied by individuals with training in mathematics education, research methodology, psychology, and psycho-linguistics. One area of concern which has emerged as a result of the interviews and analyses is symbolization, since a large part of the daily mathematical instruction in the first and second grades deals with writing, reading, and interpreting

mathematical sentences. A number of questions arise concerning this skill:

Do children connect what they write with the manipulation of real objects (beans, blocks, etc.)?

How do children interpret, in terms of mathematical sentences, the textbook pictures which are intended to suggest unique operations and sentences?

How do children view the concept of equality?

Through clinical interviews with individual children, principal investigators have attempted to find answers to these questions. These interviews suggest, for instance, that first and second graders view the symbol for equality as an operator and not as a relation symbol. For example, they accept 2 + 3 = 5 as a correct sentence, but they say that 5 = 2 + 3 is "backwards" and change it to 2 + 3 = 5. Similarly, they consider 3 = 3 as meaningless and typically change it to one of the following: 0 + 3 = 3, or 3 + 3 = 6, or 3 - 3 = 0. More generally, children seem to view mathematics as an action subject - you must do something.

When interviews revealed that first and second graders held this concept of equality, further one-to-one interviews were held with children in grades 3 through 6. Some of these interviews suggested that some children in these higher grades continue to hold this concept of equality. One further question of importance arose in connection with these findings: Does this concept of equality interfere, in a significant way, with the child's ability to grasp the concept of place value and to develop efficient computational algorithms? This question is presently being investigated by means of a teaching experiment.

Generally, the sequence of events in Project activities is something like this:

1. A principal investigator uncovers the particular way a child thinks about a specific key mathematical idea by means of an interview.
2. This investigator and other principal investigators replicate the interview.
3. If a pattern emerges, it is captured in the form of a tentative conjecture.
4. The validity of the conjecture is established by further analysis of the interview, discussion, and

reflections.

5. Ways of testing what is judged to be a significant conjecture are formulated.
6. The conjecture is tested; this may involve development of materials to be used in a teaching experiment.
7. The conjecture may become a Project finding.
8. The finding is acted upon; this may mean developing additional teaching materials and communicating the findings to the appropriate people in the profession.
9. The refined teaching materials are placed in the hands of regular teachers to be used under ordinary classroom conditions by them while being observed.

Interviewing, videotaping, and analyzing tapes are some of the ongoing activities of the Project. As of this date, taped interviews of individual children and small groups of children fall into the following eighteen categories, with the approximate times and the number of children for each category (from Tallahassee Site only):

topic	no. of children	hours
SMSG Text - Fall '74	17	5
Sense denotation	12	4
Set equivalence	8	3
Seriation	7	2
Counting	5	1
Conservation of length and measurement	34	17
Workbook exercises	16	8
Meaning of equality and addition	36	21
Missing addend	3	3
Place value	6	6
Mathematical sentences	9	9
Drill	2	1
Symbolism	34	21
KeyMath Test - Fall '75	50	19
PMDC Test - Fall '75	54	20
T1 - Multiple embodiment-teaching experiment	28	20
T3 - Equality and addition - teach. experiment	6	1
T4 - Pictures and mathematical sentences	24	12

B. Teaching Experiments and Observational Studies

As a result of interviews and observations of children and teachers, the principal investigators formulated 80 conjectures by Spring 1975. Five teaching experiments were designed to test those conjectures which seemed to have the most relevance for the learning of addition, subtraction, and place value in grades 1 and 2. The main emphases in those experiments are as follows.

1. The role of embodiments in the learning of place value, addition, and subtraction by second grade children. (To illustrate in greater detail what is meant by a teaching experiment, a copy of a teaching experiment proposal "T1, July 1975, Tallahassee Group, *Effects of the Multiple Embodiment and Mathematical Variability Principles on 2nd Graders' Learning and Understanding of Two-Digit Addition and Subtraction and Effects of Transfer to Three-Digit Addition and Subtraction,*" is enclosed.)
2. Teaching the concept of equality as a relation, rather than as an operator as is the conventional practice. The effect of the understanding and acceptance of equality as a relation upon the learning of numeration, expanded notation, bridging the tens in addition, and upon the transfer power of this understanding to addition and subtraction of two-digit numbers with regrouping is being studied.
3. Constructing and learning of alternative computational algorithms by second grade children. This teaching experiment has four objectives:
   a. To identify and study the mental operations for the learning of algorithms (particularly computational procedures) by elementary school children.
   b. To identify and study the cognitive development and readiness factors in learning whole number computation.
   c. To investigate the learning effects of instructional treatments in which it is intended that children develop and study alternative algorithms.
   d. To develop and study instructional procedures and materials for helping children generate and study alternative algorithms.
4. Investigating first grade children's degree of success

with written symbolization as a function of readiness for symbolization. The extent to which children are able to use and interpret mathematical symbols is investigated in this teaching experiment.

5. Investigating three learning and instructional phases for addition and subtraction:
   a. Exploratory phase,
   b. Abstraction-representation phase, and
   c. Formalization-interpretation phase.

Teaching materials, appropriate for each teaching experiment, are being developed. Existing manipulative aids are selected as they fit the requirements of the experiments. If the required manipulative aids do not exist, they are constructed.

The findings of each teaching experiment will be analyzed and reported. Those materials which have produced superior results will be tested by having them used by the elementary school teachers in ordinary classroom conditions.

Hopefully, one of the products of the teaching experiments will be a much better understanding of children's thinking. This should contribute some insights into the reasons for the failure of large numbers of students in understanding some basic mathematical concepts.

C. Assessment of Children's Knowledge

PMDC is developing diagnostic tests, administered on a one-to-one basis to first and second graders at the beginning and the end of the school year. Individuals are trained in the techniques of administering these instruments. Videotapes of these testing sessions are being analyzed, again with the purpose of gaining insights into children's mathematical thinking. Group data and correlations between various clusters of skills and concepts are analyzed by the use of computer programs. During Fall 1975, about 200 first and 200 second grade children were tested.

D. Dissemination

The knowledge gained by means of the three activities described above is shared with the profession, including curriculum designers and textbook writers. This is accomplished primarily through publications and professional conferences.

Each participant will receive the following materials:

1. A copy of this paper (with the teaching experiment)
2. PMDC's Newsletter No. 1
3. PMDC's Newsletter No. 2 (contains an order form for other available publications)
4. Brochure of the Conference Proceedings.

Those participants wishing to have their names placed on the mailing list for future Newsletters may give their names and mailing addresses to Dr. Nichols or send to: PMDC
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